

Hence our assumption that $T > 0$ is wrong, since hot water added is not able to melt all of the ice.

Therefore the final temperature will be 0°C .

Let m = mass of ice finally left in the can.

Heat lost by water = Heat gained by melting ice

$$30 \times 1 \times (90 - 0) = (50 - m) \times 80$$

$$\Rightarrow m = 16.25 \text{ gm.}$$

Finally there is 16.25 gm of ice and

$(200 + 30 + 33.75) = 263.75 \text{ gm}$ of water at 0°C .

Illustration - 8 If water at 0°C , kept in a container with an open top, is placed in a large evacuated chamber :

- (A) all the water will vaporize
 (B) all the water will freeze
 (C) part of the water will vaporize and the rest will freeze
 (D) ice, water and water vapour will be formed and reach equilibrium at the triple point

SOLUTION : (C)

Evacuation will greatly enhance evaporation leading to heat losses by water. As water is at 0°C , it begins to freeze. After some time, a part of water will vaporise and the remaining will freeze.

$\Rightarrow$ C is correct.

Illustration - 9 In the previous question, if the specific latent heat of vaporization of water at 0°C is η times the specific latent heat of freezing of water at 0°C , the fraction of water that will ultimately freeze is :

- (A) $\frac{1}{\eta}$ (B) $\frac{\eta}{\eta + 1}$ (C) $\frac{\eta - 1}{\eta}$ (D) $\frac{\eta - 1}{\eta + 1}$

SOLUTION : (B)

Let m_1 = mass which finally freezes

m_2 = mass of water vaporised

Heat gained by m_2 = Heat lost by m_1

$$m_2 L_{\text{vap}} = m_1 L_{\text{fusion}}$$

$$\frac{m_1}{m_2} = \eta \Rightarrow \frac{m_1}{m_1 + m_2} = \frac{\eta}{\eta + 1}$$

$\Rightarrow$ Fraction of total mass which freezes is

$$\frac{\eta}{\eta + 1}$$

B is the correct answer.

THERMAL EXPANSION

Section - 2

When a solid is heated, its length, surface area and the volume increase.

Linear expansion : (expansion in length of a solid)

Consider a rod of length ℓ_1 at a temperature T_1 . Let it be heated to a temperature T_2 and the increased length of the rod be ℓ_2 , then :

$$\ell_2 = \ell_1 (1 + \alpha \theta)$$

α : coefficient of linear expansion and $\theta = T_2 - T_1$

Superficial expansion : (expansion in surface area)

If A_1 is the area of solid at $T_1^\circ\text{C}$ and A_2 is the area at $T_2^\circ\text{C}$, then :

$$A_2 = A_1 (1 + \beta\theta) \quad \theta = T_2 - T_1$$

β : coefficient of superficial (area) expansion

Volume expansion : (expansion in volume)

(a) If V_1 is the volume of solid at $T_1^\circ\text{C}$ and V_2 is the volume at $T_2^\circ\text{C}$, then :

$$V_2 = V_1 (1 + \gamma\theta)$$

γ : coefficient of cubical (volume) expansion

(b) As the temperature increases, density of solid decreases. If d_1 is the density at $T_1^\circ\text{C}$, d_2 is the density at $T_2^\circ\text{C}$, then :

$$d_2 = \frac{d_1}{(1 + \gamma\theta)}$$

Relation between α , β and γ :

$$\beta = 2\alpha, \quad \gamma = 3\alpha \quad [6\alpha = 3\beta = 2\gamma]$$

Heating a metallic scale :

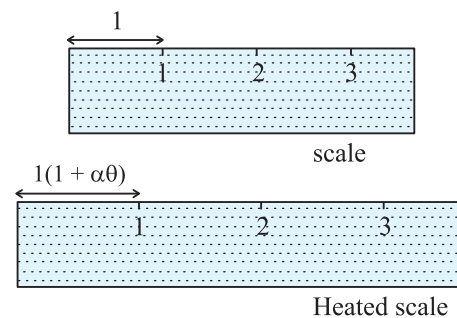
A metallic scale (linear) expands in length when heated. As a result all the markings are displaced from their usual (correct) positions.

A reading of 1 unit on a heated scale is equivalent to an actual length of $1(1 + \alpha\theta)$, where α is coefficient of linear expansion of material of scale, and θ is rise in the temperature of the scale.

$\Rightarrow$ If the reading is ℓ , actual length = $\ell(1 + \alpha\theta)$

$\Rightarrow$ If the reading is x , actual length = $x(1 + \alpha\theta)$

$\Rightarrow$ actual length = reading $(1 + \alpha\theta)$



Difference of lengths of two rods :

Consider two rods 1 and 2 of lengths ℓ_1 and ℓ_2 . Let them be heated through a temperature θ . If ℓ'_1 and ℓ'_2 are their expanded lengths, then :

$$\ell'_2 = \ell_2 (1 + \alpha_2\theta) \quad \alpha_2 : \text{coefficient of linear expansion of rod 2}$$

$$\ell'_1 = \ell_1 (1 + \alpha_1\theta) \quad \alpha_1 : \text{coefficient of linear expansion of rod 1}$$

If the difference of lengths of two rods is constant :

$$\Rightarrow \ell'_2 - \ell'_1 = \ell_2 - \ell_1$$

$$\Rightarrow \ell_1 \alpha_1 = \ell_2 \alpha_2$$

Time period of pendulum :

Time period (T) of a simple pendulum of length ℓ is given by :

$$T = 2\pi \sqrt{\frac{\ell}{g}}$$

If there is a rise in temperature by θ , length of the pendulum increases and hence the time period increases. As a result, the clock supporting such a simple pendulum slows down.

If ℓ_0 be the length of the pendulum and corresponding time period be T_0 , then : $T_0 = 2\pi \sqrt{\frac{\ell_0}{g}}$

If the pendulum be heated by θ (rise in temperature), the new time period T_t is : $T_t = 2\pi \sqrt{\frac{\ell_t}{g}}$ ℓ_t : expanded length

$$\Rightarrow \frac{T_t}{T_0} = \sqrt{\frac{\ell_t}{\ell_0}} = \sqrt{\frac{\ell_0(1+\alpha\theta)}{\ell_0}}$$

$$\Rightarrow \frac{T_t}{T_0} = \sqrt{1+\alpha\theta} = 1 + \frac{1}{2}\alpha\theta \quad \text{as } \alpha \text{ is very small} \quad \Rightarrow \quad \frac{T_t - T_0}{T_0} = \frac{1}{2}\alpha\theta \quad \Rightarrow \quad \frac{\Delta T}{T_0} = \frac{1}{2}\alpha\theta$$

$$\Rightarrow \frac{\Delta T}{T_0} = \text{fractional increase in } T = \text{time lost in 1 sec.}$$

$$\Rightarrow \text{Time lost in one day} = \left(\frac{1}{2}\alpha\theta\right) \times 3600 \times 24 \text{ s}$$

$$= \left(\frac{1}{2}\alpha\theta\right) \times 86400 \text{ s}$$

Illustration - 10 A surveyor's 30 m steel tape is correct at a temperature of 20°C . The distance between two points, as measured by this tape on a day when the temperature is 35°C , is 26 m. What is the true distance between the points ? ($\alpha_{\text{steel}} = 1.2 \times 10^{-5}/^\circ\text{C}$)

SOLUTION :

Let temperature rise above the correct temperature be θ .

$$\Rightarrow \theta = 35 - 20 = 15^\circ\text{C}.$$

Using the relation :

Correct length = measured length $(1 + \alpha\theta)$

True distance between the points

$$= 26(1 + 1.2 \times 10^{-5} \times 15)$$

$$\Rightarrow \text{true distance} = 26.00468 \text{ m.}$$

Illustration - 11 A clock with a brass pendulum shaft keeps correct time at a certain temperature.

(a) How closely must the temperature be controlled if the clock is not to gain or lose more than 1 sec. a day ? Does the answer depend on the period of the pendulum ?

(b) Will an increase of temperature cause the clock to gain or lose ? ($\alpha_{\text{brass}} = 2 \times 10^{-5}/^\circ\text{C}$)

SOLUTION :

(a) No. of seconds lost or gained per day

$$= \frac{1}{2}\alpha\theta \times 86400$$

where : θ = rise or drop in temperature

α = coeff. of linear expansion of shaft.

$$\text{We want that } \left| \frac{1}{2} \alpha \theta \times 86400 \right| < 1$$

$$\Rightarrow |\theta| < \frac{2}{2 \times 10^{-5} \times 86400}$$

$$\Rightarrow |\theta| < 1.1574^\circ\text{C}$$

Hence temperature should not increase or decrease by more than 1.1574°C . This does not depend on the time period.

- (b) An increase in temperature makes the pendulum slow and hence clock loses time.

Illustration - 12 A steel ring of 3.000 inches inside diameter at 20°C is to be heated and slipped over a brass shaft measuring 3.002 inches in diameter at 20°C . To what temperature should the ring be heated? ($\alpha_{\text{steel}} = 1.2 \times 10^{-5}/^\circ\text{C}$)

SOLUTION :

Let θ be the temperature to which the ring must be heated.

Final diameter of ring should be 3.002 inches.

$$\Rightarrow 3.002 = 3 [1 + \alpha (\theta - 20)]$$

$$\Rightarrow \theta = \frac{3.002 - 3}{3 \alpha} + 20$$

$$\Rightarrow \theta = 75.6^\circ\text{C}$$

Illustration - 13 A pendulum clock loses 12 sec. a day if the temperature is 40°C and goes fast by 4 sec. a day if the temperature is 20°C . Find the temperature at which the clock will show correct time and the coefficient of linear expansion of the metal of the pendulum shaft.

SOLUTION :

Let T be the temperature at which the clock is correct.

Time lost per day = $1/2 \alpha$ (rise in temperature) $\times 86400$

$$\Rightarrow 12 = 1/2 \alpha (40 - T) \times 86400 \quad \dots\dots(i)$$

Time gained per day

$$= 1/2 \alpha (\text{drop in temperature}) \times 86400$$

$$\Rightarrow 4 = 1/2 \alpha (T - 20) \times 86400 \quad \dots\dots(ii)$$

On adding (i) and (ii), we get :

$$64 = 86400 \alpha (40 - 20)$$

$$\Rightarrow \alpha = 1.85 \times 10^{-5}/^\circ\text{C}.$$

On dividing (i) and (ii), we get :

$$12 (T - 20) = 4 (40 - T)$$

$$\Rightarrow T = 25^\circ\text{C}.$$

$$\Rightarrow \text{Clock shows correct time at } 25^\circ\text{C}.$$

Illustration - 14 A glass flask whose volume is exactly 1000 cm^3 at 0°C is filled level full of mercury at this temperature. When the flask and mercury are heated to 100°C , 15.2 cm^3 of mercury overflow. If the coefficient of cubical expansion of Hg is $1.82 \times 10^{-4}/^\circ\text{C}$, compute the coefficient of linear expansion of glass.

SOLUTION :

As 15.2 cm^3 of Hg overflow at 100°C ,

final volume of Hg – final volume of glass flask

$$= 15.2 \text{ cm}^3$$

$$\Rightarrow 1000 (1 + \gamma_\ell \theta) - 1000 (1 + \gamma_g \theta) = 15.2$$

where θ = rise in temperature = $100 - 0 = 100^\circ\text{C}$

$$\Rightarrow \gamma_g = \gamma_\ell - \frac{15.2}{1000 \theta} = 0.000182 - 0.000152$$

$$\Rightarrow \gamma_g = 0.00003/^\circ\text{C} = 3 \times 10^{-5} (^\circ\text{C})^{-1}$$

$$\Rightarrow \alpha_g = \frac{\gamma_g}{3} = 1 \times 10^{-5} (^\circ\text{C})^{-1}$$

Illustration - 15 A 250 cm^3 glass bottle is completely filled with water at 50°C . The bottle and water are heated to 60°C . How much water runs over if :

- (a) the expansion of the bottle is neglected ?
 (b) the expansion of the bottle is included ? Given the coefficient of areal expansion of glass

$$\gamma_g = 1.2 \times 10^{-5} / ^\circ\text{C} \text{ and } \gamma_{\text{water}} = 60 \times 10^{-5} / ^\circ\text{C}.$$

SOLUTION :

Water overflow = (final volume of water)
 – (final volume of bottle)

- (a) If the expansion of bottle is neglected :

$$\begin{aligned} \text{Water overflow} &= 250 (1 + \gamma_\ell \theta) - 250 \\ &= 250 \times 60 \times 10^{-5} \times 10 \\ \Rightarrow \text{water overflow} &= 1.5 \text{ cm}^3. \end{aligned}$$

- (b) If the bottle (glass) expands :

$$\begin{aligned} \text{Water overflow} &= (\text{final volume of water}) \\ &\quad - (\text{final volume of glass}) \\ &= 250 (1 + \gamma_\ell \theta) - 250 (1 + \gamma_g \theta) = 250 (\gamma_\ell - \gamma_g) \theta \\ &= 250 (58.8 \times 10^{-5}) \times (60 - 50) \\ \Rightarrow \text{water overflow} &= 1.47 \text{ cm}^3. \end{aligned}$$

TRANSMISSION OF HEAT

Section - 3

There are three different ways in which heat can be transferred: conduction, convection and radiation.

Conduction :

It is a process by which heat is transferred in solids. In conduction, molecules vibrate about a fixed location and transfer the heat by collision.

Convection :

It is a process by which heat is transferred in fluids (liquids & gases). In convection, transfer of heat takes place by transport of matter (in form of motion of particles).

When a liquid in a container is heated, the molecules at the lower layers are heated up and the density decreases. As a result the molecules rise up and heavier ones come down and hence a continuous movement of molecules takes place giving rise to convection currents. In this manner the whole of liquid gets heated.

Radiation :

It is a process in which the heat is transferred in the form of electromagnetic waves without the aid of any material medium. Heat is radiated from a body due to its own temperature (*greater than 0 K*) is called as **radiant energy**.

Thermal Conductivity

In solids, heat is transferred through conduction. We will study conduction of heat through a solid bar in the following section.

Consider a solid bar of thickness d and area of cross-section A . The left side of the bar is maintained at a temperature θ_A and right side at θ_B .